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MODELS FOR STARS OF VERY LOW MASS

by Shiv S. Kumar
Goddard Institute for Space Studies
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SUMMARY

Completely convective models have been constructed for stars of masses 0.09, 0.08, 0.07, 0.06, 0.05, and 0.04 (solar units), taking into account the nonrelativistic degeneracy of the stellar material. The properties of these models are presented in tabular form and in the form of graphs. It is shown that there is a lower limit to the mass of a main sequence star. The stars with mass less than this limit become completely degenerate stars or "black" dwarfs as a consequence of the gravitational contraction and therefore they never go through the normal stellar evolution.

CONTENTS

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INTRODUCTION

Very little observational or theoretical information is available concerning the structure of stars of very low mass. In this paper an attempt will be made to study the internal structure of stars of mass $M < 0.1\,M_\odot$ by using suitable theoretical models. In particular we shall be concerned with the effects of degeneracy on the structure of stars having masses between $0.09\,M_\odot$ and $0.04\,M_\odot$.

THE EQUATION OF STATE FOR THE STELLAR MATERIAL

We shall use here the following equation of state for the partially degenerate matter (Tolman, 1938):

$$P = P_n + P_e = N_n kT + N_e \frac{V(\alpha, 3/2)}{V(\alpha, 1/2)} kT , \tag{1}$$

where P is the total gas pressure, P_n is the pressure due to nuclei which are assumed to be nondegenerate, P_e the pressure due to electrons, N_n the density of nuclei, N_e the density of the electrons, k the Boltzmann constant, T the temperature, and the function $V(\alpha, \rho)$ is defined by

$$V(\alpha, \rho) = \frac{1}{\Gamma(\rho + 1)} \int_0^\infty \frac{z^\rho \, dz}{e^{\alpha + z} + 1} , \tag{2}$$

where α is a parameter which is a function of N_e and the temperature, and ρ is equal to 3/2 or 1/2. The equation of state (Equation 1) is derived by using the Fermi-Dirac statistics and is valid for a mixture of nuclei and electrons in which the velocities of electrons are small as compared with the velocity of light. At high densities, the electron gas becomes degenerate but the degeneracy will be nonrelativistic in the stars of low mass because we do not expect to have densities higher than 10^4 gm/cm^3 in these objects.

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MODELS FOR STARS OF VERY LOW MASS

In order to evaluate physical quantities such as the central temperature and the central density, we have to make use of certain models for the stars under study. Now, we are primarily concerned with these stars when they are contracting or when nuclear reactions involving the destruction of H^2, Li^6, Li^7, and $B_e{}^9$ are taking place. In both cases we can assume that the models are completely convective, as has recently been shown by Hayashi (1962). Therefore, we study the structure of stars of low mass by assuming that they can be represented by spheres of polytropic index 1.5. Before we derive the expressions for the temperature, density, and pressure inside the star, let us introduce here, following Tolman's notation, the degeneracy parameter y given by

$$y = \frac{N_e h^3}{2\left(2\pi m_e kT\right)^{3/2}} \quad . \tag{3}$$

It can be shown that

$$y = V(\alpha,\ 1/2). \tag{4}$$

Denoting $\frac{V(\alpha,\ 3/2)}{V(\alpha,\ 1/2)}$ by $D(y)$, we obtain from Equation 1:

$$T = \frac{\mu M_H}{k}\frac{P}{\rho}\,\frac{N_n + N_e}{N_n + D(y)N_e} \quad , \tag{5}$$

where ρ is the density and μ is the molecular weight defined by

$$\frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{1}{2}Z \ . \tag{6}$$

Here X, Y, and Z are the abundances, by weight, of hydrogen, helium, and heavier elements, respectively. If we write

$$\Lambda(y) = \frac{N_n + N_e}{N_n + D(y)\ N_e} \ , \tag{7}$$

then the equation of state (Equation 5) takes the form

$$T = \frac{\mu M_H}{k}\frac{P}{\rho}\ \Lambda(y) \ . \tag{8}$$

This form of the equation is similar to the equation of state for the perfect gas:

$$T = \frac{\mu M_H}{k}\frac{P}{\rho} \quad . \tag{9}$$

Because of degeneracy of the electron gas, we have an additional factor $\Lambda(y)$ in Equation 8. Using the equation of state (Equation 8) and the fact that the star is in convective equilibrium we obtain

the following expressions for the temperature, density, and pressure at any point inside the star:

$$T = 1.246 \times 10^7 \mu \frac{M}{R} \Lambda(y) \theta(\xi) , \tag{10}$$

$$\rho = 8.446 \frac{M}{R^3} \theta(\xi)^{3/2} , \tag{11}$$

$$P = 8.680 \times 10^{15} \frac{M^2}{R^4} \theta(\xi)^{5/2} , \tag{12}$$

where $\theta(\xi)$ is the Lane-Emden function for the polytrope of index 1.5 and

$$\xi = \frac{r}{1.905 \times 10^{10} R} , \tag{13}$$

r being the distance in centimeters measured from the center. Here M and R are expressed in solar units. Because the models chosen for study are in convective equilibrium, the quantity $\Lambda(y)$ is a constant throughout the star. The fact that y or $\Lambda(y)$ has the same value in all regions of the star is something unique to the completely convective models, as has already been pointed out by Limber (1958) who studied completely convective models for M dwarfs. It is this fact that greatly facilitates the computation of models for stars of low mass. At the center of the star

$$T_c = 1.246 \times 10^7 \mu \frac{M}{R} \Lambda(y) , \tag{14}$$

$$\rho_c = 8.446 \frac{M}{R^3} , \tag{15}$$

$$P_c = 8.680 \times 10^{15} \frac{M^2}{R^4} . \tag{16}$$

Therefore, degeneracy of the material does not affect the central density and the central pressure while it introduces a multiplicative factor in the expression for the central temperature T_c. In order to determine T_c, we must know $\Lambda(y)$ as a function of M, R, and μ. From Equation 3 and the definition of N_e, we obtain

$$y = 6.185 \times 10^7 (1 + X) \frac{\rho}{T^{3/2}} . \tag{17}$$

Eliminating T and ρ between Equations 17, 11, and 10, we obtain

$$\Lambda(y) = \frac{5.205 \times 10^{-2} (1 + X)^{2/3}}{y^{2/3} M^{1/3} \mu R} . \tag{18}$$

Thus knowing y, we can obtain $\Lambda(y)$ for a given M, R, and μ. The equations developed here have also been used by Limber (1958) in his study of the late type main sequence stars. However, we intend to

apply this procedure not to the main sequence stars but the contracting stars of very low mass. We now apply these equations to compute the physical structure of the stars of very low mass.

COMPUTATION OF THE PHYSICAL STRUCTURE

We compute the physical structure for stars having the following two chemical compositions:

1. $X = 0.90$, $Y = 0.09$, $Z = 0.01$;

2. $X = 0.62$, $Y = 0.35$, $Z = 0.03$.

For each chemical composition and a given mass, we compute P_c , T_c, ρ_c , $\Lambda(y)$, and y at several values of the radius. The relation between y and $D(y)$ which is needed for these computations is given in Table 1. This table has been prepared by making use of the tables of the Fermi-Dirac functions computed by McDougall and Stoner (1938). For the first composition, Tables 2 to 7 give the physical quantities for $M = 0.09, 0.08, 0.07, 0.06, 0.05, 0.04$, and at several radii for each mass. No models have been computed for those radii at which electron conduction becomes very important, and consequently the assumption of convective equilibrium does not hold. For each mass, the computation of models was stopped when the degeneracy parameter y reached a value close to 15. When y reaches this value, the material becomes appreciably degenerate and only then does the electron conduction become an efficient process for heat transport. However, to obtain a rough estimate of the central density and central temperature at a smaller radius, one more model was computed for each mass by making use of the condition of convective equilibrium. When y has a value of 30 or 40, electron conduction may destroy convective equilibrium in the central regions; still the convective model should give numerical results which have the correct order of magnitude.

Table 1

The Relation Between y and $D(y)$.

y	$D(y)$	y	$D(y)$
0.000	1.0000	0.50	1.088
0.001	1.0002	0.60	1.105
0.005	1.0009	0.70	1.122
0.010	1.0017	0.80	1.140
0.015	1.0023	0.90	1.157
0.020	1.0035	1.00	1.174
0.025	1.0043	2.00	1.341
0.030	1.0053	3.00	1.504
0.035	1.0061	4.00	1.661
0.040	1.0070	5.00	1.814
0.045	1.0078	6.00	1.962
0.05	1.009	7.00	2.107
0.06	1.011	8.00	2.248
0.07	1.012	9.00	2.386
0.08	1.014	10.00	2.521
0.09	1.016	15.00	3.158
0.10	1.018	20.00	3.744
0.20	1.035	30.00	4.808
0.30	1.053	40.00	5.772
0.40	1.070		

For a given mass and chemical composition, there exists a limiting value of the radius below which there exists no model, for the material has become completely degenerate. At this stage, the star begins to approach closely a cooling curve in the H-R diagram. This limiting radius is obtained by using the asymptotic relation for $D(y)$:

$$D(y) = 0.4836\, y^{2/3} \quad (y \to \infty) . \tag{19}$$

From Equations 7 and 19, we have

$$\Lambda(y) = \frac{N_n + N_e}{N_n + 0.4836\, y^{2/3} N_e} \quad (y \to \infty) . \tag{20}$$

Table 2

Physical Properties of the Models.

X=.90 Y=.09 Z=.01 MASS=.09 MU=.534

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
5.000	0.0173	1.0000	6.081E-03	1.198E 05	1.125E 11
4.000	0.0238	1.0000	1.188E-02	1.497E 05	2.746E 11
3.000	0.0375	1.0000	2.815E-02	1.996E 05	8.680E 11
2.000	0.0690	1.0000	9.502E-02	2.994E 05	4.394E 12
1.500	0.1060	1.0000	2.252E-01	3.993E 05	1.389E 13
1.000	0.1980	0.9823	7.601E-01	5.883E 05	7.031E 13
0.900	0.2335	0.9778	1.043E 00	6.506E 05	1.072E 14
0.800	0.2800	0.9745	1.485E 00	7.295E 05	1.717E 14
0.700	0.3450	0.9691	2.216E 00	8.291E 05	2.928E 14
0.600	0.4380	0.9645	3.519E 00	9.627E 05	5.425E 14
0.500	0.5840	0.9552	6.081E 00	1.144E 06	1.125E 15
0.400	0.8500	0.9298	1.188E 01	1.392E 06	2.746E 15
0.300	1.3750	0.8989	2.815E 01	1.794E 06	8.680E 15
0.200	3.0900	0.7972	9.502E 01	2.387E 06	4.394E 16
0.190	3.3900	0.7780	1.108E 02	2.452E 06	5.395E 16
0.180	3.8300	0.7574	1.303E 02	2.520E 06	6.698E 16
0.170	4.3900	0.7322	1.547E 02	2.579E 06	8.418E 16
0.160	5.0800	0.7059	1.856E 02	2.642E 06	1.073E 17
0.150	6.1000	0.6662	2.252E 02	2.660E 06	1.389E 17
0.140	7.3700	0.6295	2.770E 02	2.693E 06	1.830E 17
0.130	9.3500	0.5784	3.460E 02	2.665E 06	2.462E 17
0.120	12.5000	0.5163	4.399E 02	2.577E 06	3.391E 17
0.115	15.0000	0.4770	4.998E 02	2.484E 06	4.020E 17
0.100	32.2000	0.3288	7.601E 02	1.969E 06	7.031E 17

THE LIMITING RADIUS=.0819

Table 3

Physical Properties of the Models.

X=.90 Y=.09 Z=.01 MASS=.08 MU=.534

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
5.000	0.0184	1.0000	5.405E-03	1.065E 05	8.888E 10
4.000	0.0258	1.0000	1.056E-02	1.331E 05	2.170E 11
3.000	0.0396	1.0000	2.503E-02	1.774E 05	6.858E 11
2.000	0.0725	1.0000	8.446E-02	2.662E 05	3.472E 12
1.500	0.1130	1.0000	2.002E-01	3.549E 05	1.097E 13
1.000	0.2100	0.9822	6.757E-01	5.229E 05	5.555E 13
0.900	0.2480	0.9769	9.269E-01	5.778E 05	8.467E 13
0.800	0.2980	0.9724	1.320E 00	6.471E 05	1.356E 14
0.700	0.3670	0.9672	1.970E 00	7.355E 05	2.314E 14
0.600	0.4670	0.9611	3.128E 00	8.527E 05	4.286E 14
0.500	0.6200	0.9548	5.405E 00	1.017E 06	8.888E 14
0.400	0.9100	0.9240	1.056E 01	1.230E 06	2.170E 15
0.300	1.4700	0.8948	2.503E 01	1.588E 06	6.858E 15
0.200	3.2800	0.7858	8.446E 01	2.092E 06	3.472E 16
0.190	3.7100	0.7622	9.851E 01	2.136E 06	4.263E 16
0.180	4.2000	0.7407	1.159E 02	2.191E 06	5.292E 16
0.170	4.8100	0.7165	1.375E 02	2.244E 06	6.651E 16
0.160	5.6200	0.6862	1.650E 02	2.283E 06	8.477E 16
0.150	6.7500	0.6478	2.002E 02	2.299E 06	1.097E 17
0.140	8.3000	0.6048	2.462E 02	2.300E 06	1.446E 17
0.130	10.7500	0.5481	3.075E 02	2.244E 06	1.945E 17
0.120	14.8000	0.4798	3.910E 02	2.128E 06	2.679E 17
0.100	41.0000	0.2917	6.757E 02	1.553E 06	5.555E 17

THE LIMITING RADIUS=.0852

Table 4

Physical Properties of the Models.

X=.90 Y=.09 Z=.01 MASS=.07 MU=.534

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
4.000	0.0275	1.0000	9.238E-03	1.164E 05	1.661E 11
3.000	0.0423	1.0000	2.190E-02	1.553E 05	5.251E 11
2.000	0.0780	1.0000	7.390E-02	2.329E 05	2.658E 12
1.500	0.1210	0.9890	1.752E-01	3.071E 05	8.401E 12
1.000	0.2250	0.9811	5.912E-01	4.570E 05	4.253E 13
0.900	0.2660	0.9749	8.110E-01	5.046E 05	6.483E 13
0.800	0.3190	0.9715	1.155E 00	5.656E 05	1.038E 14
0.700	0.3940	0.9646	1.724E 00	6.419E 05	1.771E 14
0.600	0.5010	0.9588	2.737E 00	7.443E 05	3.282E 14
0.500	0.6780	0.9403	4.730E 00	8.760E 05	6.805E 14
0.400	0.9800	0.9195	9.238E 00	1.071E 06	1.661E 15
0.300	1.6000	0.8845	2.190E 01	1.373E 06	5.251E 15
0.200	3.5800	0.7752	7.390E 01	1.805E 06	2.658E 16
0.190	4.1000	0.7455	8.620E 01	1.828E 06	3.264E 16
0.180	4.6700	0.7215	1.014E 02	1.867E 06	4.052E 16
0.170	5.4000	0.6936	1.203E 02	1.900E 06	5.092E 16
0.160	6.3900	0.6585	1.443E 02	1.917E 06	6.490E 16
0.150	7.7000	0.6203	1.752E 02	1.926E 06	8.401E 16
0.140	9.6500	0.5718	2.155E 02	1.902E 06	1.107E 17
0.130	12.7500	0.5114	2.691E 02	1.832E 06	1.489E 17
0.125	15.0000	0.4759	3.027E 02	1.773E 06	1.742E 17
0.115	23.4000	0.3768	3.887E 02	1.526E 06	2.432E 17

THE LIMITING RADIUS=.0891

Table 5

Physical Properties of the Models.

X=.90 Y=.09 Z=.01 MASS=.06 MU=.534

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
3.000	0.0460	1.0000	1.877E-02	1.331E 05	3.858E 11
2.000	0.0844	1.0000	6.334E-02	1.996E 05	1.953E 12
1.500	0.1310	0.9876	1.502E-01	2.629E 05	6.172E 12
1.000	0.2430	0.9810	5.068E-01	3.917E 05	3.125E 13
0.900	0.2875	0.9745	6.951E-01	4.323E 05	4.763E 13
0.800	0.3470	0.9670	9.898E-01	4.826E 05	7.629E 13
0.700	0.4270	0.9624	1.477E 00	5.489E 05	1.301E 14
0.600	0.5470	0.9520	2.346E 00	6.335E 05	2.411E 14
0.500	0.7300	0.9424	4.054E 00	7.525E 05	5.000E 14
0.400	1.0600	0.9183	7.918E 00	9.166E 05	1.221E 15
0.300	1.7500	0.8767	1.877E 01	1.167E 06	3.858E 15
0.210	3.6700	0.7646	5.472E 01	1.454E 06	1.607E 16
0.200	4.0800	0.7481	6.334E 01	1.493E 06	1.953E 16
0.190	4.6400	0.7229	7.388E 01	1.519E 06	2.398E 16
0.180	5.3100	0.6971	8.689E 01	1.546E 06	2.977E 16
0.170	6.2100	0.6650	1.031E 02	1.562E 06	3.741E 16
0.160	7.4000	0.6289	1.237E 02	1.569E 06	4.768E 16
0.150	9.1000	0.5843	1.502E 02	1.555E 06	6.172E 16
0.140	11.7000	0.5295	1.847E 02	1.510E 06	8.134E 16
0.135	13.7000	0.4956	2.060E 02	1.466E 06	9.406E 16
0.120	24.7000	0.3754	2.933E 02	1.249E 06	1.507E 17

THE LIMITING RADIUS=.0938

Table 6

Physical Properties of the Models.

X-.90   Y=.09   Z=.01   MASS=.05   MU=.534

RACIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
3.000	0.0500	1.0000	1.564E-02	1.109E 05	2.479E 11
2.000	0.0925	1.0000	5.279E-02	1.664E 05	1.356E 12
1.500	0.1440	0.9851	1.251E-01	2.185E 05	4.286E 12
1.000	0.2670	0.9790	4.223E-01	3.257E 05	2.170E 13
0.900	0.3160	0.9722	5.793E-01	3.594E 05	3.307E 13
0.800	0.3810	0.9653	8.248E-01	4.015E 05	5.298E 13
0.700	0.4690	0.9607	1.231E 00	4.566E 05	9.038E 13
0.600	0.6100	0.9406	1.955E 00	5.216E 05	1.674E 14
0.500	0.8100	0.9344	3.378E 00	6.218E 05	3.472E 14
0.400	1.1700	0.9144	6.598E 00	7.606E 05	8.477E 14
0.300	2.0000	0.8526	1.564E 01	9.456E 05	2.679E 15
0.210	4.2100	0.7415	4.560E 01	1.175E 06	1.116E 16
0.200	4.7800	0.7148	5.279E 01	1.189E 06	1.356E 16
0.190	5.5000	0.6871	6.157E 01	1.203E 06	1.665E 16
0.180	6.2000	0.6681	7.241E 01	1.235E 06	2.067E 16
0.170	7.4000	0.6289	8.596E 01	1.231E 06	2.598E 16
0.160	8.9000	0.5900	1.031E 02	1.227E 06	3.311E 16
0.150	11.3000	0.5373	1.251E 02	1.192E 06	4.286E 16
0.140	15.0000	0.4754	1.539E 02	1.130E 06	5.649E 16
0.120	35.0000	0.3162	2.444E 02	8.767E 05	1.046E 17

THE LIMITING RADIUS=.0997

Table 7

Physical Properties of the Models.

X=.90 Y=.C9 Z=.01 MASS=.04 MU=.534

RACIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
2.000	0.1040	0.9882	4.223E-02	1.315E 05	8.680E 11
1.500	0.1610	0.9848	1.001E-01	1.747E 05	2.743E 12
1.000	0.3000	0.9759	3.378E-01	2.598E 05	1.389E 13
0.900	0.3550	0.9691	4.634E-01	2.866E 05	2.117E 13
0.800	0.4270	0.9640	6.598E-01	3.207E 05	3.391E 13
0.700	0.5290	0.9551	9.850E-01	3.632E 05	5.784E 13
0.600	0.6830	0.9397	1.564E 00	4.169E 05	1.072E 14
0.500	0.9100	0.9313	2.703E 00	4.958E 05	2.222E 14
0.400	1.3400	0.8996	5.279E 00	5.986E 05	5.425E 14
0.300	2.3000	0.8370	1.251E 01	7.426E 05	1.715E 15
0.250	3.3700	0.7780	2.162E 01	8.283E 05	3.555E 15
0.240	3.6900	0.7630	2.444E 01	8.462E 05	4.186E 15
0.230	4.0600	0.7470	2.777E 01	8.645E 05	4.963E 15
0.220	4.5000	0.7293	3.173E 01	8.823E 05	5.929E 15
0.210	5.0500	0.7072	3.648E 01	8.964E 05	7.141E 15
0.200	5.8000	0.6747	4.223E 01	8.979E 05	8.680E 15
0.190	6.6500	0.6510	4.925E 01	9.120E 05	1.066E 16
0.180	7.8000	0.6176	5.793E 01	9.133E 05	1.323E 16
0.170	9.4000	0.5775	6.876E 01	9.042E 05	1.663E 16
0.160	11.7000	0.5303	8.248E 01	8.822E 05	2.119E 16
0.155	13.3000	0.5027	9.072E 01	8.632E 05	2.406E 16
0.140	22.2000	0.3965	1.231E 02	7.538E 05	3.615E 16

THE LIMITING RADIUS=.1074

It is now a straightforward procedure to obtain the limiting radius from Equations 18 and 20. The limiting radii obtained in this way agree with those computed from the mass-radius relation for completely degenerate configurations. In Figures 1 to 6, ρ_c and T_c have been plotted as a function of radius. (The figures follow the text.) As the radius of the star is decreased, the central density keeps on increasing while the temperature T_c shows an interesting variation. At first it increases and then reaches a maximum value. As the radius is further decreased, the temperature begins to decrease. This behavior of the temperature is due to the effects of degeneracy. We can visualize it physically in this way: When the radius changes from a large value to smaller ones, the star of a given mass can be pictured as a contracting star and only a part of the energy released by gravitational contraction is absorbed by the stellar material. However, when the material has become partially degenerate, the energy absorbed does not manifest itself as thermal energy. When partially degenerate gas is compressed, energy is needed to bring the degenerate electrons closer and therefore the central temperature remains constant for a while and later it begins to decrease as a result of further contraction.

For the central temperature, the solid curve represents those models in which electron conduction is supposed to be negligible. We have assumed that as long as y remains equal to or less than 15 the stars under study can be represented accurately by completely convective models. The dotted part of the R-T_c curve represents models for which y is greater than 15 and therefore this section of the curve is not too accurate. The solid vertical line in each graph gives the limiting radius for a given mass. Table 8 gives the values of radius R, T_c, and ρ_c corresponding to the stage of maximum central temperature. The values of ρ_c and R for the stage of complete degeneracy are given in Table 9.

In order to show the difference between the convective models composed of partially degenerate matter and of perfect gas we have computed the variation of temperature inside a star of mass 0.07 and radius 0.5. In Table 10 are given the temperature distributions for the two cases. It also gives the density distribution and pressure distribution which are the same for both cases. In Figure 7, these results are shown graphically where x, the distance from the center of the star, is defined by $x = 0.2737\,\xi$.

Table 8

Properties at Maximum Central Temperature.

Mass	X = 0.90 Y = 0.09 Z = 0.01			X = 0.62 Y = 0.35 Z = 0.03		
	Radius	Temperature	Density	Radius	Temperature	Density
0.09	0.14	2.693×10^6	2.770×10^2	0.11	4.420×10^6	5.711×10^2
0.08	0.14	2.300×10^6	2.462×10^2	0.11	3.761×10^6	5.076×10^2
0.07	0.15	1.926×10^6	1.752×10^2	0.12	3.151×10^6	3.421×10^2
0.06	0.16	1.569×10^6	1.237×10^2	0.12	2.574×10^6	2.933×10^2
0.05	0.18	1.235×10^6	7.241×10^1	0.13	2.018×10^6	1.922×10^2
0.04	0.18	9.133×10^5	5.793×10^1	0.14	1.500×10^6	1.231×10^2

Table 9

Properties of the Completely Degenerate Models.

Mass	X = 0.90 Y = 0.09 Z = 0.01		X = 0.62 Y = 0.35 Z = 0.03	
	Limiting Radius	Limiting Density	Limiting Radius	Limiting Density
0.09	0.0819	1.384×10^3	0.0628	3.069×10^3
0.08	0.0852	1.092×10^3	0.0653	2.427×10^3
0.07	0.0891	8.357×10^2	0.0683	1.856×10^3
0.06	0.0938	6.141×10^2	0.0719	1.363×10^3
0.05	0.0997	4.261×10^2	0.0764	9.471×10^2
0.04	0.1074	2.726×10^2	0.0823	6.059×10^2

Table 10

Temperature, Density, and Pressure Distributions.

X = 0.90 Y = 0.09 Z = 0.01 μ = 0.534 Mass = 0.07 Radius = 0.5

$\frac{r}{R}$	Temperature		Density	Pressure
	Perfect Gas	Degenerate Gas		
0.0000	9.316×10^5	8.760×10^5	4.730	6.805×10^{14}
0.0547	9.255×10^5	8.702×10^5	4.683	6.693×10^{14}
0.1095	9.071×10^5	8.530×10^5	4.545	6.366×10^{14}
0.1642	8.772×10^5	8.248×10^5	4.322	5.430×10^{14}
0.2190	8.369×10^5	7.869×10^5	4.027	5.204×10^{14}
0.2737	7.874×10^5	7.404×10^5	3.675	4.469×10^{14}
0.3284	7.304×10^5	6.868×10^5	3.284	3.703×10^{14}
0.3832	6.676×10^5	6.277×10^5	2.869	2.958×10^{14}
0.4379	6.007×10^5	5.648×10^5	2.449	2.272×10^{14}
0.4926	5.317×10^5	4.999×10^5	2.039	1.674×10^{14}
0.5474	4.620×10^5	4.344×10^5	1.675	1.179×10^{14}
0.6295	3.595×10^5	3.380×10^5	1.134	6.295×10^{13}
0.6842	2.943×10^5	2.767×10^5	8.400×10^{-1}	3.817×10^{13}
0.7390	2.326×10^5	2.187×10^5	5.903×10^{-1}	2.120×10^{13}
0.7937	1.750×10^5	1.646×10^5	3.853×10^{-1}	1.103×10^{13}
1.0000	0.000	0.000	0.000	0.000

In Tables 11 to 16, physical quantities are tabulated for the second chemical composition and Figures 1 to 6 show the variation of T_c and ρ_c as a function of radius. Since the central density is independent of the chemical composition, the $R - \rho_c$ curve for a given mass is the same for the two compositions considered here. The broken curve in each of these graphs shows the $R - T_c$ relation for the second composition. The dotted part of this curve has the same significance as the dotted part of the curve for the first composition. The broken vertical line shows the limiting radius for a model of a given mass having the second composition. For this composition the properties of the stars at the stage of maximum central temperature are given in Table 8, whereas those corresponding to the stage of complete degeneracy are given in Table 9. The temperature and density distribution for a star of mass 0.07 and radius 0.5 for this chemical composition are given in Table 17 and they are shown graphically in Figure 8. It is interesting to plot the central density — central temperature relation for a given mass in the temperature-density diagram in which the degenerate region can be separated from the nondegenerate one. For the first composition Figure 9 shows the positions of various models in such a diagram. For a given mass, the log ρ_c - log T_c gives an evolutionary path for a contracting star. Figure 10 shows the same graph for the second chemical composition. The boundary between the nondegenerate and degenerate regions is obtained by equating the electron pressure from the perfect gas law to that obtained from the equation of state for a completely degenerate gas.

DISCUSSION

The numerical results presented here show clearly that for a given composition, there exists a limiting mass below which a contracting star cannot reach the main sequence stage, because the temperature and density at the center are too low for hydrogen burning to start. Instead the star becomes a degenerate star as a consequence of the contraction. After the star has evolved beyond the stage of maximum central temperature, further contraction will take it towards the stage of complete degeneracy. Thus, all stars having a mass less than a certain limiting mass ultimately become completely degenerate objects without ever going through the normal stellar evolution. The exact determination of this limiting mass for a given composition requires a knowledge of the luminosity of the contracting stars which can be obtained if we know the atmospheric structure in addition to the interior models computed here. Suitable model atmospheres for contracting stars of low mass are being computed which, together with the interior models presented here, will give us not only the evolutionary tracks in the H-R diagram for these stars, but also the limiting mass which gives a lower limit to the mass of a main sequence star and the time scale for the Helmholtz-Kelvin contraction. If we assume reasonable luminosities for these stars, we find that, for stars with population I composition, the limiting mass is approximately 0.07. Similarly for the population II stars the limiting mass is approximately 0.09.

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Table 11

Physical Properties of the Models.

X=.62 Y=.35 Z=.03 MASS=.09 MU=.659

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
5.000	0.0105	1.0000	6.081E-03	1.478E 05	1.125E 11
4.000	0.0150	1.0000	1.188E-02	1.847E 05	2.746E 11
3.000	0.0231	1.0000	2.815E-02	2.463E 05	8.680E 11
2.000	0.0427	1.0000	9.502E-02	3.695E 05	4.394E 12
1.500	0.0660	1.0000	2.252E-01	4.927E 05	1.389E 13
1.000	0.1218	1.0000	7.601E-01	7.390E 05	7.031E 13
0.900	0.1430	0.9880	1.043E 00	8.112E 05	1.072E 14
0.800	0.1715	0.9851	1.485E 00	9.100E 05	1.717E 14
0.700	0.2110	0.9802	2.216E 00	1.035E 06	2.928E 14
0.600	0.2665	0.9787	3.519E 00	1.205E 06	5.425E 14
0.500	0.3550	0.9701	6.081E 00	1.434E 06	1.125E 15
0.400	0.5080	0.9548	1.188E 01	1.764E 06	2.746E 15
0.300	0.8180	0.9267	2.815E 01	2.283E 06	8.680E 15
0.200	1.6600	0.8673	9.502E 01	3.205E 06	4.394E 16
0.150	2.9000	0.7970	2.252E 02	3.926E 06	1.389E 17
0.140	3.3900	0.7696	2.770E 02	4.062E 06	1.830E 17
0.130	4.0200	0.7366	3.460E 02	4.187E 06	2.462E 17
0.120	4.8900	0.7035	4.399E 02	4.332E 06	3.391E 17
0.110	6.2000	0.6580	5.711E 02	4.420E 06	4.802E 17
0.100	8.3000	0.5932	7.601E 02	4.384E 06	7.031E 17
0.098	8.9300	0.5770	8.076E 02	4.351E 06	7.623E 17
0.096	9.6000	0.5616	8.592E 02	4.323E 06	8.278E 17
0.090	12.2500	0.5089	1.043E 03	4.179E 06	1.072E 18
0.080	22.5000	0.3810	1.485E 03	3.519E 06	1.717E 18

THE LIMITING RADIUS=.0628

Table 12

Physical Properties of the Models.

X=.62 Y=.35 Z=.03 MASS=.08 MU=.659

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
5.000	0.0115	1.0000	5.405E-03	1.314E 05	8.888E 10
4.000	0.0160	1.0000	1.056E-02	1.642E 05	2.170E 11
3.000	0.0244	1.0000	2.503E-02	2.190E 05	6.858E 11
2.000	0.0453	1.0000	8.446E-02	3.284E 05	3.472E 12
1.500	0.0700	1.0000	2.002E-01	4.379E 05	1.097E 13
1.000	0.1295	1.0000	6.757E-01	6.569E 05	5.555E 13
0.900	0.1520	0.9867	9.269E-01	7.201E 05	8.467E 13
0.800	0.1820	0.9841	1.320E 00	8.080E 05	1.356E 14
0.700	0.2240	0.9797	1.970E 00	9.193E 05	2.314E 14
0.600	0.2840	0.9755	3.128E 00	1.068E 06	4.286E 14
0.500	0.3780	0.9675	5.405E 00	1.271E 06	8.888E 14
0.400	0.5400	0.9536	1.056E 01	1.566E 06	2.170E 15
0.300	0.8700	0.9251	2.503E 01	2.026E 06	6.858E 15
0.200	1.7700	0.8647	8.446E 01	2.840E 06	3.472E 16
0.150	3.1800	0.7806	2.002E 02	3.418E 06	1.097E 17
0.130	4.4000	0.7257	3.075E 02	3.667E 06	1.945E 17
0.120	5.4500	0.6800	3.910E 02	3.722E 06	2.679E 17
0.110	6.9800	0.6299	5.076E 02	3.761E 06	3.794E 17
0.100	9.5500	0.5620	6.757E 02	3.692E 06	5.555E 17
0.099	9.9000	0.5542	6.964E 02	3.677E 06	5.783E 17
0.098	10.2500	0.5491	7.179E 02	3.680E 06	6.023E 17
0.090	14.6000	0.4691	9.269E 02	3.424E 06	8.467E 17
0.080	28.9000	0.3363	1.320E 03	2.761E 06	1.356E 18

THE LIMITING RADIUS=.0653

Table 13

Physical Properties of the Models.

X=.62 Y=.35 Z=.03 MASS=.07 MU=.659

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
5.000	0.0125	1.0000	4.730E-03	1.150E 05	6.805E 10
4.000	0.0170	1.0000	9.238E-03	1.437E 05	1.661E 11
3.000	0.0262	1.0000	2.190E-02	1.916E 05	5.251E 11
2.000	0.0483	1.0000	7.390E-02	2.874E 05	2.658E 12
1.500	0.0748	1.0000	1.752E-01	3.832E 05	8.401E 12
1.000	0.1382	0.9892	5.912E-01	5.686E 05	4.253E 13
0.900	0.1628	0.9852	8.110E-01	6.292E 05	6.483E 13
0.800	0.1950	0.9828	1.155E 00	7.061E 05	1.038E 14
0.700	0.2400	0.9780	1.724E 00	8.030E 05	1.771E 14
0.600	0.3050	0.9726	2.737E 00	9.317E 05	3.282E 14
0.500	0.4050	0.9660	4.730E 00	1.110E 06	6.805E 14
0.400	0.5810	0.9493	9.238E 00	1.364E 06	1.661E 15
0.300	0.9350	0.9218	2.190E 01	1.766E 06	5.251E 15
0.200	1.9400	0.8502	7.390E 01	2.443E 06	2.658E 16
0.150	3.5000	0.7649	1.752E 02	2.931E 06	8.401E 16
0.140	4.1000	0.7379	2.155E 02	3.029E 06	1.107E 17
0.130	4.9400	0.7014	2.691E 02	3.101E 06	1.489E 17
0.120	6.1500	0.6579	3.421E 02	3.151E 06	2.051E 17
0.110	8.0500	0.5980	4.442E 02	3.125E 06	2.905E 17
0.100	11.2000	0.5288	5.912E 02	3.039E 06	4.253E 17
0.098	12.2500	0.5091	6.282E 02	2.986E 06	4.611E 17
0.095	14.2000	0.4741	6.896E 02	2.868E 06	5.222E 17
0.080	40.0000	0.2825	1.155E 03	2.030E 06	1.038E 18

THE LIMITING RADIUS=.0683

Table 14

Physical Properties of the Models.

X=.62 Y=.35 Z=.03 MASS=.06 MU=.659

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
4.000	0.0186	1.0000	7.918E-03	1.232E 05	1.221E 11
3.000	0.0284	1.0000	1.877E-02	1.642E 05	3.858E 11
2.000	0.0521	1.0000	6.334E-02	2.463E 05	1.953E 12
1.500	0.0810	1.0000	1.502E-01	3.284E 05	6.172E 12
1.000	0.1500	0.9862	5.068E-01	4.859E 05	3.125E 13
0.900	0.1760	0.9847	6.951E-01	5.390E 05	4.763E 13
0.800	0.2110	0.9819	9.898E-01	6.047E 05	7.629E 13
0.700	0.2600	0.9762	1.477E 00	6.870E 05	1.301E 14
0.600	0.3300	0.9715	2.346E 00	7.977E 05	2.411E 14
0.500	0.4400	0.9623	4.054E 00	9.482E 05	5.000E 14
0.400	0.6330	0.9440	7.918E 00	1.163E 06	1.221E 15
0.300	1.0250	0.9053	1.877E 01	1.487E 06	3.858E 15
0.200	2.1500	0.8355	6.334E 01	2.058E 06	1.953E 16
0.180	2.6400	0.8094	8.689E 01	2.215E 06	2.977E 16
0.150	3.9300	0.7424	1.502E 02	2.438E 06	6.172E 16
0.140	4.6800	0.7100	1.847E 02	2.498E 06	8.134E 16
0.130	5.7000	0.6712	2.307E 02	2.544E 06	1.094E 17
0.120	7.1800	0.6270	2.933E 02	2.574E 06	1.507E 17
0.110	9.5500	0.5624	3.807E 02	2.519E 06	2.134E 17
0.100	14.0000	0.4800	5.068E 02	2.365E 06	3.125E 17
0.090	24.8000	0.3639	6.951E 02	1.992E 06	4.763E 17

THE LIMITING RADIUS=.0719

Table 15

Physical Properties of the Models.

X=.62 Y=.35 Z=.03 MASS=.05 MU=.659

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
4.000	0.0202	1.0000	6.598E-03	1.026E 05	8.477E 10
3.000	0.0310	1.0000	1.564E-02	1.368E 05	2.679E 11
2.000	0.0575	1.0000	5.279E-02	2.053E 05	1.356E 12
1.500	0.0886	1.0000	1.251E-01	2.737E 05	4.286E 12
1.000	0.1645	0.9854	4.223E-01	4.045E 05	2.170E 13
0.900	0.1930	0.9841	5.793E-01	4.489E 05	3.307E 13
0.800	0.2315	0.9807	8.248E-01	5.033E 05	5.298E 13
0.700	0.2850	0.9758	1.231E 00	5.723E 05	9.038E 13
0.600	0.3630	0.9688	1.955E 00	6.629E 05	1.674E 14
0.500	0.4850	0.9584	3.378E 00	7.869E 05	3.472E 14
0.400	0.7080	0.9309	6.598E 00	9.554E 05	8.477E 14
0.300	1.1400	0.9038	1.564E 01	1.237E 06	2.679E 15
0.200	2.4000	0.8249	5.279E 01	1.693E 06	1.356E 16
0.180	3.0000	0.7899	7.241E 01	1.802E 06	2.067E 16
0.160	3.9100	0.7426	1.031E 02	1.905E 06	3.311E 16
0.150	4.5500	0.7171	1.251E 02	1.963E 06	4.286E 16
0.140	5.5000	0.6794	1.539E 02	1.992E 06	5.649E 16
0.130	6.7500	0.6390	1.922E 02	2.018E 06	7.598E 16
0.120	8.7000	0.5814	2.444E 02	1.989E 06	1.046E 17
0.110	12.0000	0.5132	3.173E 02	1.915E 06	1.482E 17
0.090	38.0000	0.2909	5.793E 02	1.327E 06	3.307E 17

THE LIMITING RADIUS=.0764

Table 16

Physical Properties of the Models.

X=.62 Y=.35 Z=.03 MASS=.04 MU=.659

RADIUS	Y	LAMBDA(Y)	DENSITY(C)	TEMP(C)	PRESSURE(C)
3.000	0.0350	1.0000	1.251E-02	1.095E 05	1.715E 11
2.000	0.0642	1.0000	4.223E-02	1.642E 05	8.680E 11
1.500	0.0995	0.9893	1.001E-01	2.166E 05	2.743E 12
1.000	0.1840	0.9852	3.378E-01	3.236E 05	1.389E 13
0.900	0.2160	0.9833	4.634E-01	3.588E 05	2.117E 13
0.800	0.2590	0.9803	6.598E-01	4.025E 05	3.391E 13
0.700	0.3200	0.9731	9.850E-01	4.566E 05	5.784E 13
0.600	0.4090	0.9639	1.564E 00	5.276E 05	1.072E 14
0.500	0.5460	0.9540	2.703E 00	6.267E 05	2.222E 14
0.400	0.7930	0.9298	5.279E 00	7.634E 05	5.425E 14
0.300	1.3000	0.8917	1.251E 01	9.762E 05	1.715E 15
0.250	1.8000	0.8615	2.162E 01	1.132E 06	3.555E 15
0.200	2.8000	0.8017	4.223E 01	1.317E 06	8.680E 15
0.190	3.1300	0.7836	4.925E 01	1.355E 06	1.066E 16
0.180	3.5400	0.7629	5.793E 01	1.392E 06	1.323E 16
0.170	4.0300	0.7378	6.876E 01	1.425E 06	1.663E 16
0.160	4.6900	0.7114	8.248E 01	1.460E 06	2.119E 16
0.150	5.5500	0.6764	1.001E 02	1.481E 06	2.743E 16
0.140	6.7500	0.6393	1.231E 02	1.500E 06	3.615E 16
0.130	8.5900	0.5836	1.538E 02	1.474E 06	4.863E 16
0.120	11.4500	0.5216	1.955E 02	1.428E 06	6.698E 16
0.100	30.5000	0.3187	3.378E 02	1.047E 06	1.389E 17

THE LIMITING RADIUS=.0823

Table 17

Temperature, Density and Pressure Distributions.

X = 0.62 Y = 0.35 Z = 0.03 μ = 0.659 Mass = 0.07 Radius = 0.5

$\frac{r}{R}$	Temperature		Density	Pressure
	Perfect Gas	Degenerate Gas		
0.0000	1.149×10^6	1.110×10^6	4.730	6.805×10^{14}
0.0547	1.141×10^6	1.103×10^6	4.683	6.693×10^{14}
0.1095	1.119×10^6	1.081×10^6	4.545	6.366×10^{14}
0.1642	1.082×10^6	1.045×10^6	4.322	5.430×10^{14}
0.2190	1.032×10^5	9.971×10^5	4.027	5.204×10^{14}
0.2737	9.711×10^5	9.382×10^5	3.675	4.469×10^{14}
0.3284	9.008×10^5	8.702×10^5	3.284	3.703×10^{14}
0.3832	8.234×10^5	7.954×10^5	2.869	2.958×10^{14}
0.4379	7.409×10^5	7.157×10^5	2.449	2.272×10^{14}
0.4926	6.557×10^5	6.335×10^5	2.039	1.674×10^{14}
0.5474	5.698×10^5	5.504×10^5	1.675	1.179×10^{14}
0.6295	4.434×10^5	4.283×10^5	1.134	6.295×10^{13}
0.6842	3.630×10^5	3.506×10^5	8.400×10^{-1}	3.817×10^{13}
0.7390	2.869×10^5	2.772×10^5	5.903×10^{-1}	2.120×10^{13}
0.7937	2.159×10^5	2.086×10^5	3.853×10^{-1}	1.103×10^{13}
1.0000	0.000	0.000	0.000	0.000

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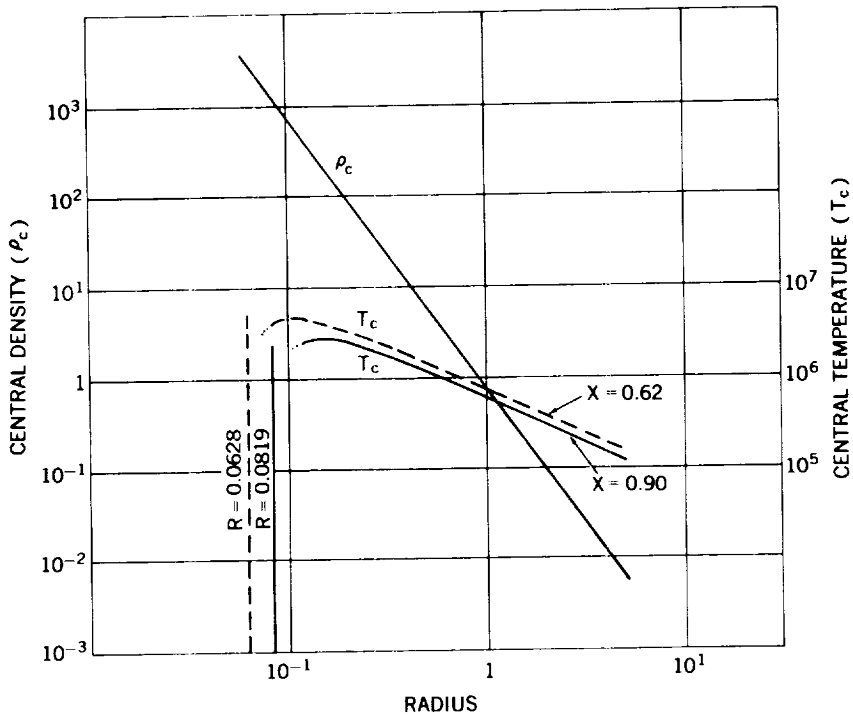

Figure 1—The central temperature and the central density in completely convective models. Mass = 0.09.

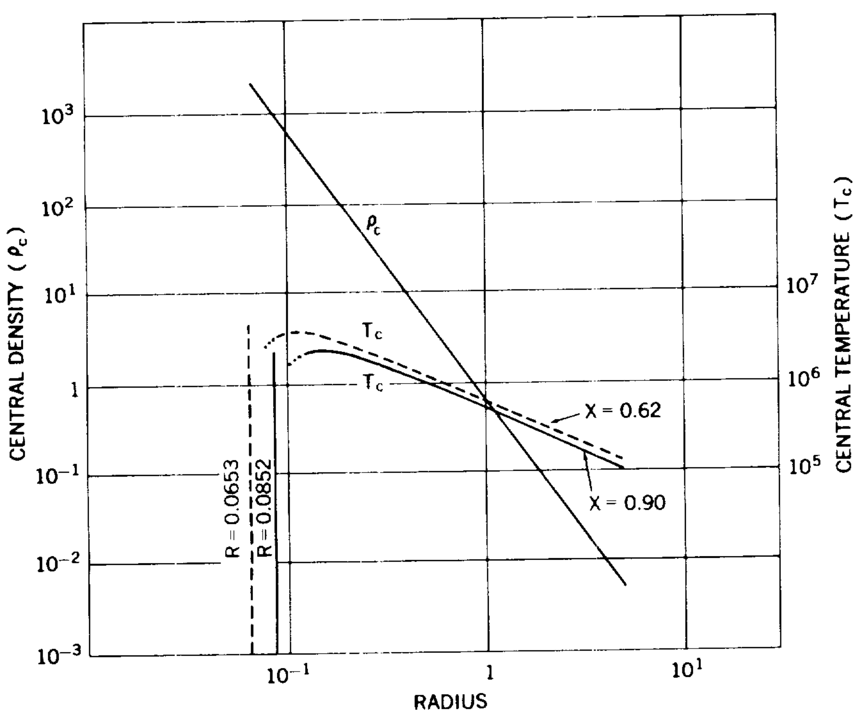

Figure 2—The central temperature and the central density in completely convective models. Mass = 0.08.

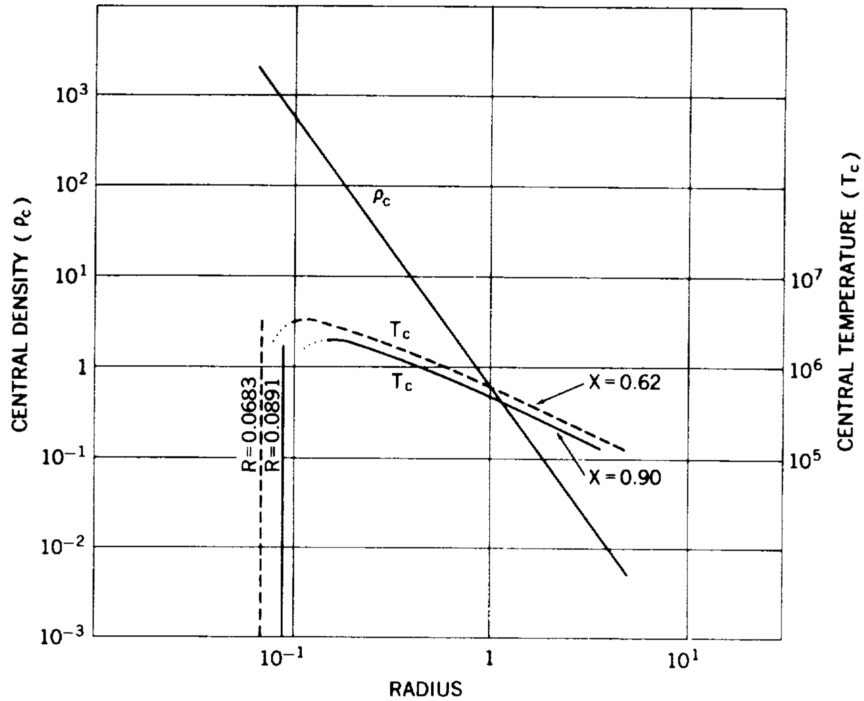

Figure 3—The central temperature and the central density in completely convective models. Mass = 0.07.

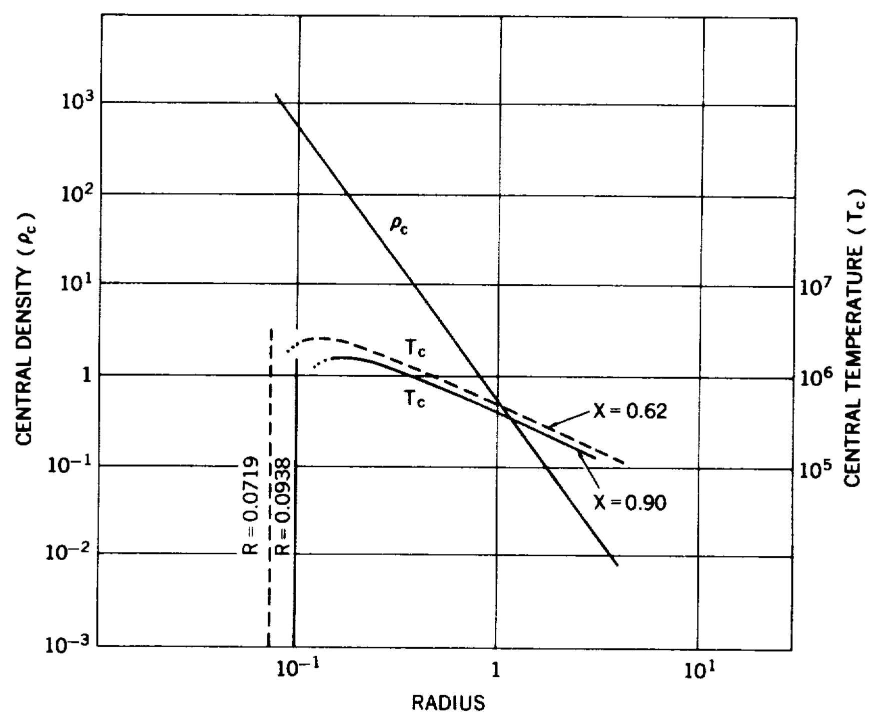

Figure 4—The central temperature and the central density in completely convective models. Mass = 0.06.

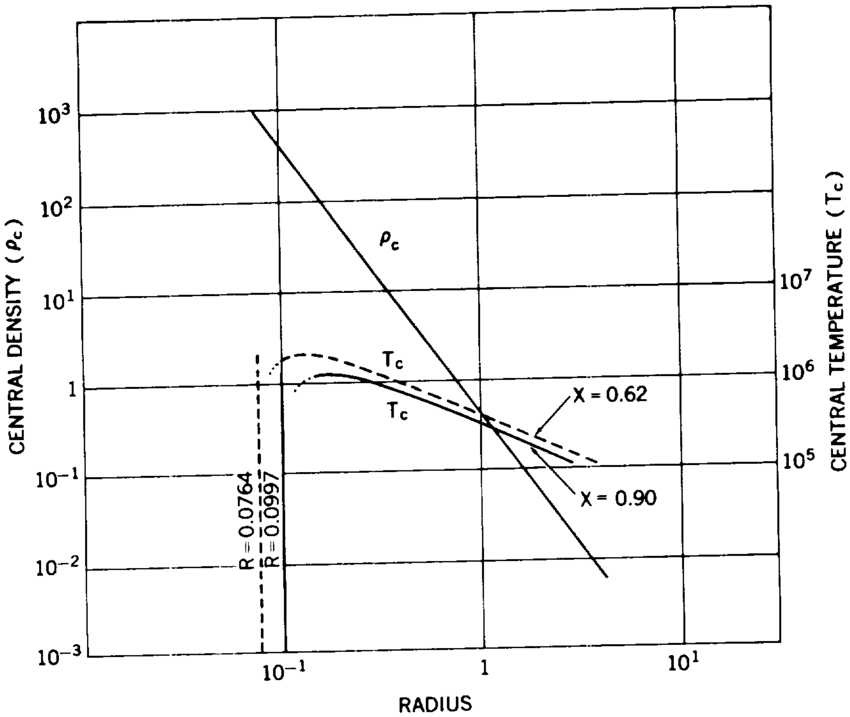

Figure 5—The central temperature and the central density in completely convective models. Mass = 0.05.

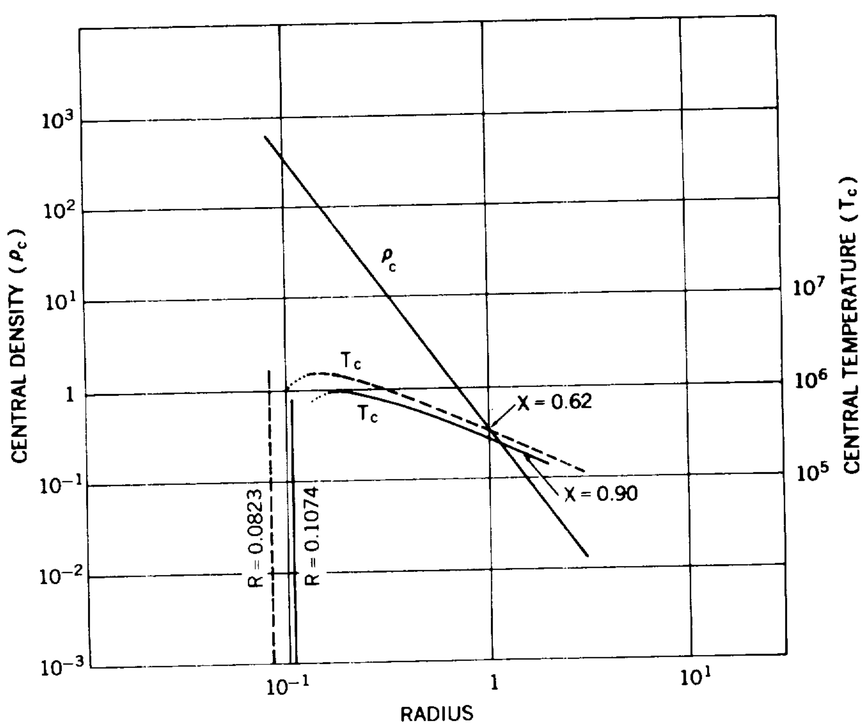

Figure 6—The central temperature and the central density in completely convective models. Mass = 0.04.

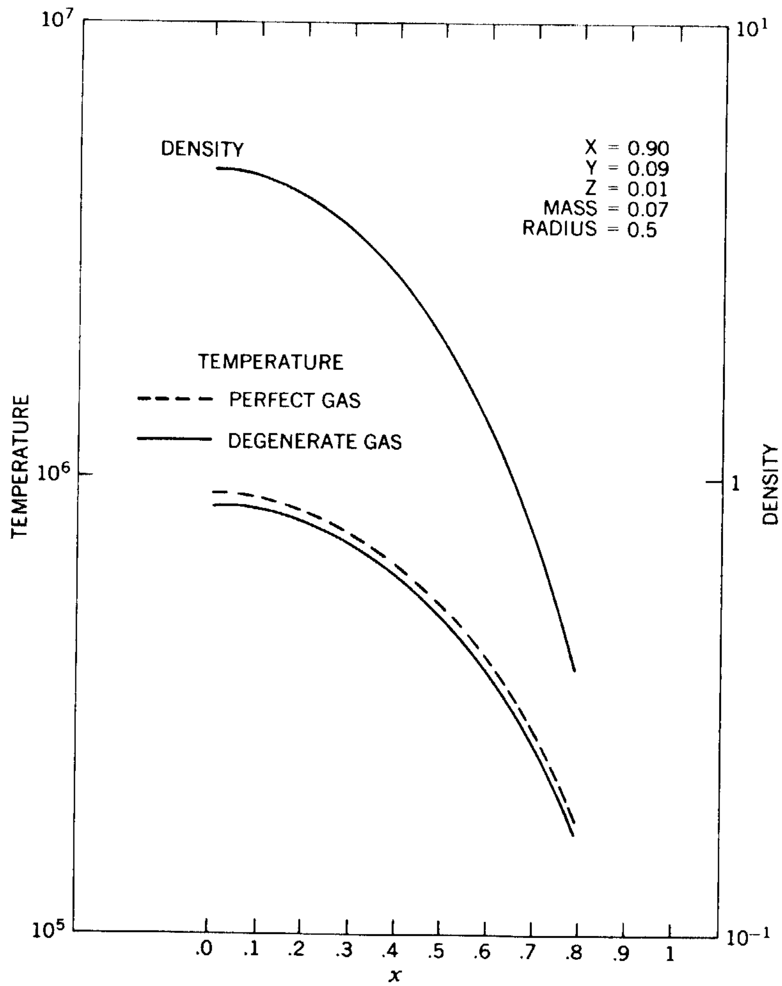

Figure 7—The temperature and density distributions in a completely convective model. Population II.

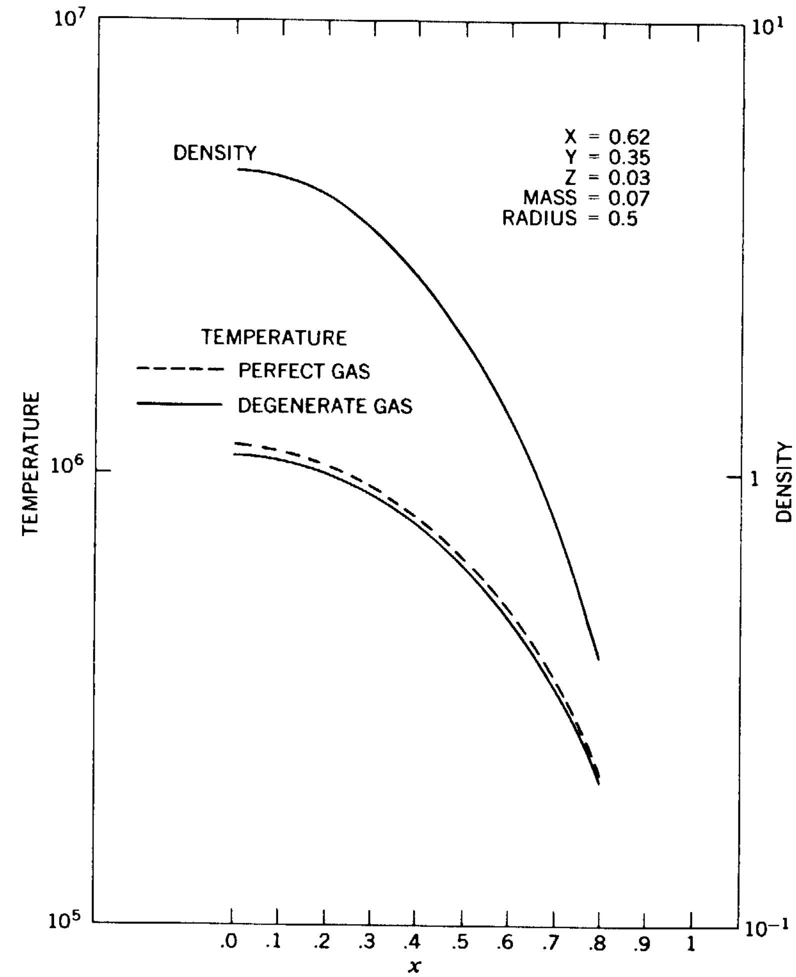

Figure 8—The temperature and density distributions in a completely convective model. Population I.

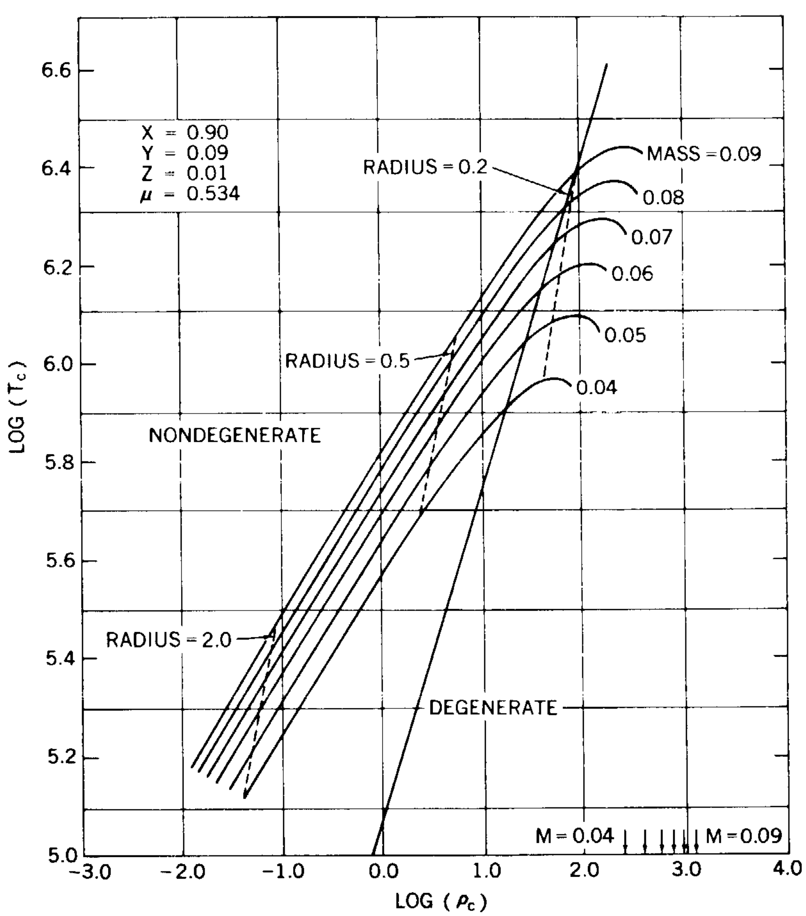

Figure 9—The temperature-density diagram for completely convective models. Population II.

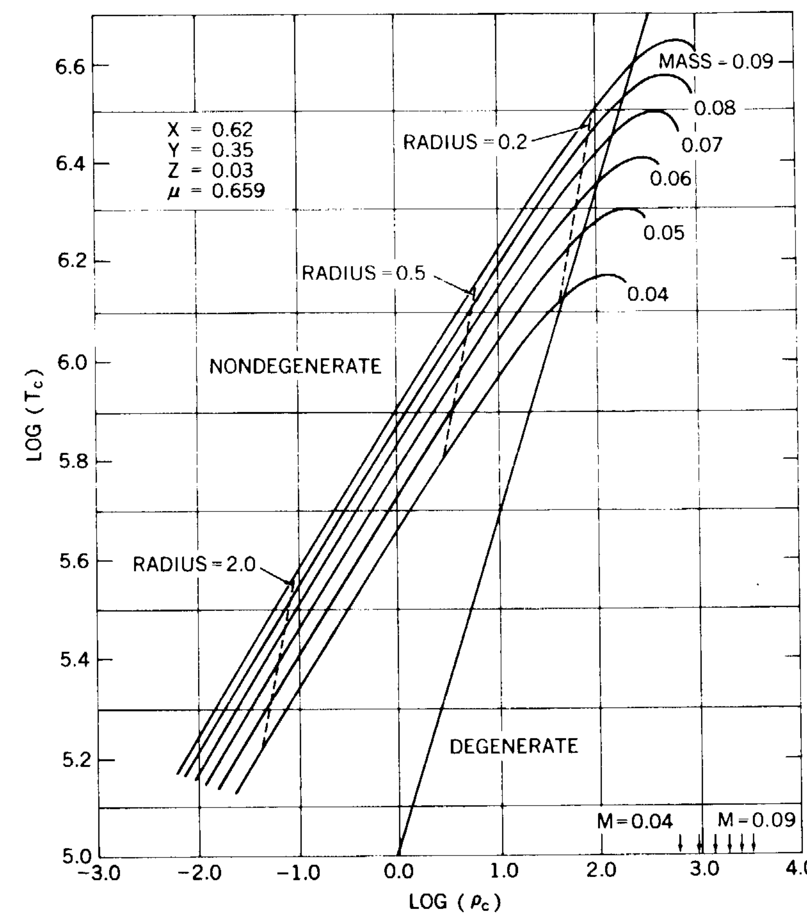

Figure 10—The temperature-density diagram for completely convective models. Population I.